Differentiating Exponentials and Logarithms

Objective: by the end of this lesson you should be able to use the chain, product and quotient rules when differentiating exponentials and logarithms

Rules for exponentials

$$y = ae^{x} \qquad \frac{dy}{dx} = ae^{x}$$

$$y = e^{ax} \qquad \frac{dy}{dx} = ae^{ax}$$

$$y = e^{f(x)} \qquad \frac{dy}{dx} = f'(x)e^{f(x)}$$

Rules for logarithms

$y = \ln x \quad \frac{dy}{dx} = \frac{1}{x}$

- $y = a \ln x$ $\qquad \dfrac{dy}{dx} = \dfrac{a}{x}$

- $y = \ln(ax)$ $\qquad \dfrac{dy}{dx} = \dfrac{1}{x}$

* $y = \ln(f(x))$ $\qquad \dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

Example 1

- Differentiate $y=e^{2x}$

$$\frac{dy}{dx} = 2e^{2x}$$

$4e^x$

Example 2

- Differentiate $y=4e^{(3x-1)}$

$\frac{dy}{dx} = 12e^{(3x-1)}$

Example 3

- Differentiate $y=x^2e^{3x}$

$u = x^2 \quad v = e^{3x}$

$du = 2x \quad dv = 3e^{3x}$

$\frac{dy}{dx} = 2xe^{3x} + 3x^2e^{3x}$

$\frac{dy}{dx} = xe^{3x}(2+3x)$

Example 4

Differentiate $y=5\ln x$

$$\frac{dy}{dx} = \frac{5}{x}$$

Example 5

$y=\ln(f(x))$

Differentiate $y=\ln(2x^3)$

$$\frac{dy}{dx}=\frac{6x^2}{2x^3}=\frac{3}{x}$$

Example 6

$y = \frac{u}{v}$

Differentiate $y = \frac{2\ln x}{x^2}$

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$u = 2\ln x \qquad v = x^2$

$du = \frac{2}{x} \qquad dv = 2x$

$$\frac{dy}{dx} = \frac{\frac{2x^2}{x} - 4x\ln x}{x^4} = \frac{2x - 4x\ln x}{x^4} = \frac{2x(1-2\ln x)}{x^4}$$

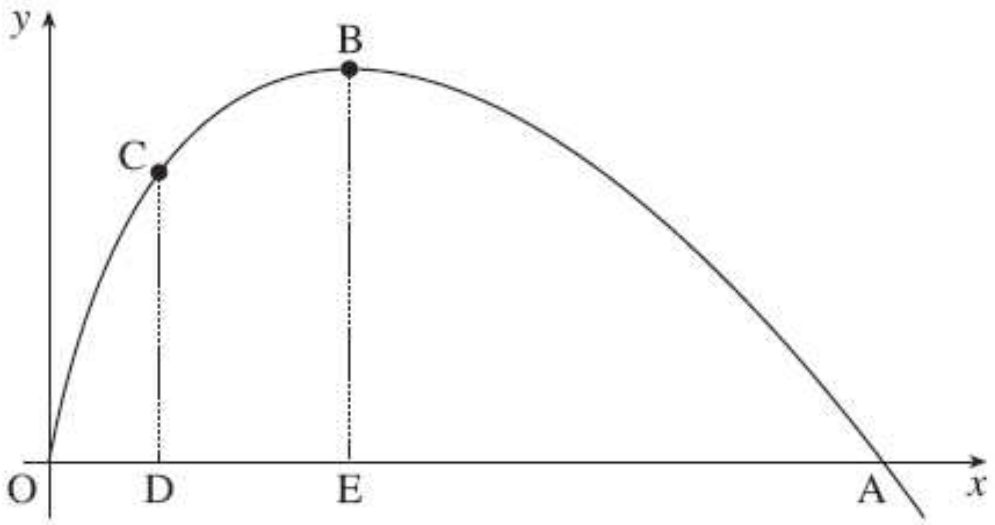
$$= \frac{2(1-2\ln x)}{x^3}$$

Exam Question

Fig. 7 shows the curve

$$y = 2x - x \ln x, \text{ where } x > 0.$$

The curve crosses the x-axis at A, and has a turning point at B. The point C on the curve has x-coordinate 1. Lines CD and BE are drawn parallel to the y-axis.



Not to scale

Fig. 7

(i) Find the x-coordinate of A, giving your answer in terms of e. [2]

(ii) Find the exact coordinates of B. [6]

(iii) Show that the tangents at A and C are perpendicular to each other. [3]

7(i) $2x - x\ln x = 0$ $\Rightarrow$ $x(2 - \ln x) = 0$ $\Rightarrow$ $(x = 0)$ or $\ln x = 2$ $\Rightarrow$ at A, $x = e^2$	M1 A1 [2]
(ii) $\frac{dy}{dx} = 2 - x.\frac{1}{x} - \ln x.1$ $= 1 - \ln x$ $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow$ $\ln x = 1, x = e$ When $x = e$, $y = 2e - e\ln e = e$ So B is (e, e)	M1 B1 A1 M1 A1cao B1ft [6]
(iii) At A, $\frac{dy}{dx} = 1 - \ln e^2 = 1 - 2$ $= -1$ At C, $\frac{dy}{dx} = 1 - \ln 1 = 1$ $1 \times -1 = -1 \Rightarrow$ tangents are perpendicular	M1 A1cao E1 [3]

Work

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