DEPARTMENT OF MATHEMATICS

Introduction to A level Mathematics

BRIDGING WORK

SUMMER 2023

Welcome to Mathematics at Heckmondwike Grammar School.

The mathematics department is staffed by eleven experienced and well-qualified mathematics teachers led by the Subject Leader, Mrs Hepworth.

Our exam board for A level is AQA.

In year 12, you will have 5 hours of lessons per week. You will have three hours of lessons in Pure Mathematics with one teacher and two hours of lessons with a different teacher for Applied Mathematics. (Mechanics and Statistics).

Students taking both Mathematics and Further Mathematics will have a total of $4\,\frac{1}{2}$ hours of Pure Mathematics & Statistics lessons with one teacher, and $4\frac{1}{2}$ hours of Pure Mathematics & Mechanics per week with another teacher.

The AS-level and A-level Mathematics courses contain a good deal of algebra, and it is important that you start your course confident in using all the techniques of algebraic manipulation that you learnt in your GCSE.

- Expanding brackets
- Simplifying expressions
- Solving equations – linear, quadratic, simultaneous
- Factorising
- Completing the square
- Changing the subject of a formula

These examples and exercises have been collected together for you to practice and review these basic techniques.

Write your solutions on lined paper. Mark and correct your work, where necessary using the answers at the end of the booklet using green pen.

You will be asked to bring in your solutions for the work in this booklet during the second week back In September.

EXPANDING BRACKETS

To remove a single bracket, we multiply every term in the bracket by the number or the expression on the outside:

Examples

1) $3(x + 2y) = 3x + 6y$

2) $-2(2x - 3) = (-2)(2x) + (-2)(-3)$
$= -4x + 6$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including

* the smiley face method
* FOIL (Fronts Outers Inners Lasts)
* using a grid.

Examples:

1) $(x + 1)(x + 2) = x(x + 2) + 1(x + 2)$

or

$(x + 1)(x + 2) = x^2 + 2 + 2x + x$
$= x^2 + 3x + 2$

or

	x	1
x	x^2	x
2	$2x$	2

$(x + 1)(x + 2) = x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

2) $(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$
$= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

or

$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$

or

	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

EXERCISE A Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a - 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $-3x - (x + 4)$
6. $5(2x - 1) - (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x - 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$
12. $(3 + 5x)(4 - x)$
13. $(x + 2)(x + 3)(x + 4)$
14. $(x - 3)^3$
15. $(2x + 3)(3x - 5)(x - 7)$
16. The x^2 term in the expansion of $(3x + 4)(x^2 + px + 5)$ is $-23x^2$
    Work out the value of p.

Two Special Cases

Perfect Square:	**Difference of two squares:**
$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$	$(x - a)(x + a) = x^2 - a^2$
$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$	$(x - 3)(x + 3) = x^2 - 3^2$
	$= x^2 - 9$

EXERCISE B Multiply out

1. $(x - 1)^2$
2. $(3x + 5)^2$
3. $(7x - 2)^2$
4. $(x + 2)(x - 2)$
5. $(3x + 1)(3x - 1)$
6. $(5y - 3)(5y + 3)$

LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in x. A linear equation does not contain any x^2 or x^3 terms.

Example 1: Solve the equation $64 - 3x = 25$

Solution: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the x term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

Example 2: Solve the equation $6x + 7 = 5 - 2x$.

Solution:
Step 1: Begin by adding $2x$ to both sides $8x + 7 = 5$
(to ensure that the x terms are together on the same side)

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -\frac{1}{4}$

Example 3: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $6x - 4 = 20 - 3x - 6$
(taking care of the negative signs)

Step 2: Simplify the right hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

Exercise C: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$ 2) $5x - 2 = 13$ 3) $11 - 4x = 5$

4) $5 - 7x = -9$ 5) $11 + 3x = 8 - 2x$ 6) $7x + 2 = 4x - 5$

Exercise D: Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 4: Solve the equation $\frac{y}{2} + 5 = 11$

Solution:

Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10: $y = 12$

Example 5: Solve the equation $\frac{1}{3}(2x + 1) = 5$

Solution:

Step 1: Multiply by 3 (to remove the fraction) $2x + 1 = 15$

Step 2: Subtract 1 from each side $2x = 14$

Step 3: Divide by 2 $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

Example 6: Solve the equation $\frac{x+1}{4} + \frac{x+2}{5} = 2$

Solution:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4} + \frac{20(x+2)}{5} = 40$

Step 3: Simplify the left hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}} + \frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}} = 40$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

Example 7: Solve the equation $x + \frac{x-2}{4} = 2 - \frac{3-5x}{6}$

Solution: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x + \frac{12(x-2)}{4} = 24 - \frac{12(3-5x)}{6}$$

Simplify	$12x + 3(x-2) = 24 - 2(3-5x)$
Expand brackets	$12x + 3x - 6 = 24 - 6 + 10x$
Simplify	$15x - 6 = 18 + 10x$
Subtract $10x$	$5x - 6 = 18$
Add 6	$5x = 24$
Divide by 5	$x = 4.8$

Exercise E: Solve these equations

1) $\frac{1}{2}(x+3) = 5$

2) $\frac{2x}{3} - 1 = \frac{x}{3} + 4$

3) $\frac{y}{4} + 3 = 5 - \frac{y}{3}$

4) $\frac{x-2}{7} = 2 + \frac{3-x}{14}$

5) $\frac{7x-1}{2} = 13 - x$

6) $\frac{y-1}{2} + \frac{y+1}{3} = \frac{2y+5}{6}$

7) $2x + \frac{x-1}{2} = \frac{5x+3}{3}$

8) $2 - \frac{5}{x} = \frac{10}{x} - 1$

FORMING EQUATIONS

Example 8: Find three consecutive numbers so that their sum is 96.

Solution: Let the first number be n, then the second is $n + 1$ and the third is $n + 2$.

Therefore $n + (n + 1) + (n + 2) = 96$

$3n + 3 = 96$

$3n = 93$

$n = 31$

So the numbers are 31, 32 and 33.

Exercise F:

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.
Form an equation, letting n be the number of photographs one girl had at the **beginning**.
Hence find how many each has **now**.

SIMULTANEOUS EQUATIONS

An example of a pair of simultaneous equations is $3x + 2y = 8$ (1)
$5x + y = 11$ (2)

In these equations, x and y stand for two numbers. We can solve these equations in order to find the values of x and y by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate y. We do this by making the coefficients of y the same in both equations. This can be achieved by multiplying equation (2) by 2, so that both equations contain $2y$:

$3x + 2y = 8$ (1)
$10x + 2y = 22$ $2 \times (2) = (3)$

To eliminate the y terms, we subtract equation (3) from equation (1). We get: $7x = 14$
i.e. $x = 2$

To find y, we substitute $x = 2$ into one of the original equations. For example if we put it into (2):

$10 + y = 11$
$y = 1$

Therefore the solution is $x = 2$, $y = 1$.

Remember: You can <u>check</u> your solutions by substituting both x and y into the original equations.

Example: Solve $2x + 5y = 16$ (1)
$3x - 4y = 1$ (2)

Solution: We begin by getting the same number of x or y appearing in both equation. We can get $20y$ in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$8x + 20y = 64$ (3)
$15x - 20y = 5$ (4)

As the SIGNS in front of $20y$ are DIFFERENT, we can eliminate the y terms from the equations by ADDING: $23x = 69$ (3) + (4)
i.e. $x = 3$

Substituting this into equation (1) gives:

$6 + 5y = 16$
$5y = 10$

So… $y = 2$

The solution is $x = 3$, $y = 2$.

Exercise G: Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) $3p + 3q = 15$
$2p + 5q = 14$

FACTORISING

Common factors

We can factorise some expressions by taking out a common factor.

Example 1: Factorise $12x - 30$

Solution: 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:

$$12x - 30 = 6(2x - 5)$$

Example 2: Factorise $6x^2 - 2xy$

Solution: 2 is a common factor to both 6 and 2. Both terms also contain an x.
So we factorise by taking $2x$ outside a bracket.

$$6x^2 - 2xy = 2x(3x - y)$$

Example 3: Factorise $9x^3y^2 - 18x^2y$

Solution: 9 is a common factor to both 9 and 18.
The highest power of x that is present in both expressions is x^2.
There is also a y present in both parts.
So we factorise by taking $9x^2y$ outside a bracket:

$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

Example 4: Factorise $3x(2x - 1) - 4(2x - 1)$

Solution: There is a common bracket as a factor.
So we factorise by taking $(2x - 1)$ out as a factor.
The expression factorises to $(2x - 1)(3x - 4)$

Exercise H

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$
The method is:
<u>Step 1</u>: Form two brackets $(x \;\ldots\;)(x \;\ldots\;)$
<u>Step 2</u>: Find two numbers that multiply to give c and add to make b. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 – 9x – 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.
Therefore $x^2 – 9x – 10 = (x – 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$
The method is:
<u>Step 1</u>: Find two numbers that multiply together to make ac and add to make b.
<u>Step 2</u>: Split up the bx term using the numbers found in step 1.
<u>Step 3</u>: Factorise the front and back pair of expressions as fully as possible.
<u>Step 4</u>: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x – 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x – 12 = \underbrace{6x^2 - 8x} + \underbrace{9x – 12}$

$= 2x(3x – 4) + 3(3x – 4)$ (the two brackets must be identical)

$= (3x – 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x – a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$

$$16x^2 - 25 = (4x)^2 - 5^2 = (2x + 5)(2x - 5)$$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x+4)(x-4)$

and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x+4y)(x-4y)$

Factorising by pairing
We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) – 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x – 1)$

Exercise I

Factorise

1) $x^2 - x - 6$	2) $x^2 + 6x - 16$	3) $2x^2 + 5x + 2$
4) $2x^2 - 3x$	5) $3x^2 + 5x - 2$	6) $2y^2 + 17y + 21$
7) $7y^2 - 10y + 3$	8) $10x^2 + 5x - 30$	9) $x^2 - 3x - xy + 3y^2$
10) $9x^2 - 25$	11) $4x^2 - 12x + 8$	12) $16m^2 - 81n^2$
13) $4y^3 - 9a^2 y$	14) $8(x+1)^2 - 2(x+1) - 10$	15) $6(2x-3)^2 + 3(3x+4) - 48$

CHANGING THE SUBJECT OF A FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

Example 1: Make x the subject of the formula $y = 4x + 3$.

Solution: $y = 4x + 3$

Subtract 3 from both sides: $y - 3 = 4x$

Divide both sides by 4; $\frac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with x the subject.

Example 2: Make x the subject of $y = 2 - 5x$

Solution: Notice that in this formula the x term is negative.

$y = 2 - 5x$

Add $5x$ to both sides $y + 5x = 2$ (the x term is now positive)

Subtract y from both sides $5x = 2 - y$

Divide both sides by 5 $x = \frac{2-y}{5}$

Example 3: The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make F the subject.

$$C = \frac{5(F-32)}{9}$$

Multiply by 9 $9C = 5(F-32)$ (this removes the fraction)

Expand the brackets $9C = 5F - 160$

Add 160 to both sides $9C + 160 = 5F$

Divide both sides by 5 $\frac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \frac{9C+160}{5}$.

Exercise J

Make x the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \dfrac{x+5}{4}$

3) $4y = \dfrac{x}{3} - 2$

4) $y = \dfrac{4(3x-5)}{9}$

Rearranging equations involving squares and square roots

Example 4: Make x the subject of $x^2 + y^2 = w^2$

Solution: $x^2 + y^2 = w^2$

Subtract y^2 from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving x)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

Example 5: Make a the subject of the formula $t = \dfrac{1}{4}\sqrt{\dfrac{5a}{h}}$

Solution: $t = \dfrac{1}{4}\sqrt{\dfrac{5a}{h}}$

Multiply by 4 $4t = \sqrt{\dfrac{5a}{h}}$

Square both sides $16t^2 = \dfrac{5a}{h}$

Multiply by h: $16t^2 h = 5a$

Divide by 5: $\dfrac{16t^2 h}{5} = a$

Exercise K

Make t the subject of each of the following

1) $P = \dfrac{wt}{32r}$

2) $P = \dfrac{wt^2}{32r}$

3) $V = \dfrac{1}{3}\pi t^2 h$

4) $P = \sqrt{\dfrac{2t}{g}}$

5) $Pa = \dfrac{w(v-t)}{g}$

6) $r = a + bt^2$

More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

Example 6: Make t the subject of the formula $a - xt = b + yt$

Solution: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add xt to both sides: $a = b + yt + xt$

Now put the terms without a t on the left hand side:

Subtract b from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by $(y + x)$: $\frac{a-b}{y+x} = t$

So the required equation is $t = \frac{a-b}{y+x}$

Example 7: Make W the subject of the formula $T - W = \frac{Wa}{2b}$

Solution: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by $2b$: $2bT - 2bW = Wa$

Add $2bW$ to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \frac{2bT}{a+2b}$

Exercise L

Make x the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \frac{2x+3}{5x-2}$

4) $\frac{x}{a} = 1 + \frac{x}{b}$

5) $xy = \frac{5x+4}{3}$

6) $a = \sqrt{\frac{b+2x}{3}}$

7) $y = \sqrt{\frac{x+4}{3x}}$

8) $y = \frac{3x^3+w}{x^3-2}$

Completing the square

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2+\frac{b}{a}x\right)+c$
$= 2\left(x^2 - \frac{5}{2}x\right) + 1$	**2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$
$= 2\left[\left(x-\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$	
$= 2\left(x-\frac{5}{4}\right)^2 - \frac{25}{8} + 1$	**3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2
$= 2\left(x-\frac{5}{4}\right)^2 - \frac{17}{8}$	**4** Simplify

Exercise M

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

c $x^2 - 8x$ **d** $x^2 + 6x$

e $x^2 - 2x + 7$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$

3 Complete the square.

a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

4 Write $7 - 12x - 18x^2$ in the form $a - 2(bx + c)^2$

SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are three methods that are commonly used for solving quadratic equations:
* factorising * the quadratic formula * completing the square

Method : Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of x^2 is positive.

Example : Solve $x^2 - 3x + 2 = 0$

Factorise $(x - 1)(x - 2) = 0$
Either $(x - 1) = 0$ or $(x - 2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

Method: Completing the Square

Example : Solve $x^2 - 2x - 1 = 0$

Write in form $(x - p)^2 + q = 0$ $(x - 1)^2 - 2 = 0$
Rearrange to make x the subject: $(x - 1)^2 = 2$
$x - 1 = \pm\sqrt{2}$
So $x = 1 \pm\sqrt{2}$

Method: Quadratic Formula

Example : Solve $3x^2 - 8x - 7 = 0$ $x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$

Write down values of a, b, c $a = 3, b = -8, c = -7$

Substitute into formula: $x = \frac{8 \pm\sqrt{(-8)^2-4\times3\times(-7)}}{2\times3}$

Simplify: $x = \frac{8 \pm\sqrt{148}}{6}$

Evaluate: $x = 3.36, -1.53$ (3.s.f.)

Exercise N

1) Use factorisation to solve the following equations:
a) $x^2 + 3x + 2 = 0$ b) $x^2 - 3x - 4 = 0$ c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:
a) $x^2 + 3x = 0$ b) $x^2 - 4x = 0$ c) $4 - x^2 = 0$

3) Solve the following equations by factorising.
a) $2x^2 + 7x + 6 = 0$ b) $8x^2 - 24x + 10 = 0$ c) $6x^2 - 5x - 4 = 0$

4) Solve the following equations by completing the square. Give your answers in surd form.
a) $x^2 - 6x - 4 = 0$ b) $x^2 + 8x + 13 = 0$ c) $2x^2 - 24x + 10 = 0$

5) Solve the following equations by quadratic formula. Round your answers to 3 significant figures.
a) $x^2 + 7x + 5 = 0$ b) $x^2 - 5x - 8 = 0$ c) $3x^2 + 5 = 9x$

6) Solve $\frac{3}{x-2} + \frac{2}{x-1} = 5$ Do not use trial and improvement. Write your solutions to 3 sig. figs.

SOLUTIONS TO THE EXERCISES

EXPANDING BRACKETS

Ex A

1) $28x + 35$ 2) $-15x + 21$ 3) $-7a + 4$ 4) $6y + 3y^2$ 5) $-4x - 4$
6) $7x - 1$ 7) $x^2 + 5x + 6$ 8) $t^2 - 7t - 10$ 9) $6x^2 + xy - 12y^2$
10) $4x^2 + 4x - 24$ 11) $4y^2 - 1$ 12) $12 + 17x - 5x^2$ 13) $x^3 + 9x^2 + 26x + 24$
14) $x^3 - 9x^2 + 27x - 27$ 15) $6x^3 - 43x^2 - 8x - 105$ 16) -9

Ex B

1) $x^2 - 2x + 1$ 2) $9x^2 + 30x + 25$ 3) $49x^2 - 28x + 4$ 4) $x^2 - 4$
5) $9x^2 - 1$ 6) $25y^2 - 9$

LINEAR EQUATIONS

Ex C

1) 7 2) 3 3) 1½ 4) 2 5) -3/5 6) -7/3

Ex D

1) 2.4 2) 5 3) 1 4) ½

Ex E

1) 7 2) 15 3) 24/7 4) 35/3 5) 3 6) 2 7) 9/5 8) 5

Ex F

1) 34, 36, 38 2) 9.875, 29.625 3) 24, 48

SIMULTANEOUS EQUATIONS

Ex G

1) $x = 1, y = 3$ 2) $x = -3, y = 1$ 3) $x = 0, y = -2$ 4) $x = 3, y = 1$
5) $a = 7,\ b = -2$ 6) $p = 11/3,\ q = 4/3$

FACTORISING

Ex H

1) $x(3 + y)$ 2) $2x(2x - y)$ 3) $pq(q - p)$ 4) $3q(p - 3q)$ 5) $2x^2(x - 3)$ 6) $4a^3b^2(2a^2 - 3b^2)$
7) $(y - 1)(5y + 3)$

Ex I

1) $(x - 3)(x + 2)$ 2) $(x + 8)(x - 2)$ 3) $(2x + 1)(x + 2)$ 4) $x(2x - 3)$
5) $(3x - 1)(x + 2)$ 6) $(2y + 3)(y + 7)$ 7) $(7y - 3)(y - 1)$ 8) $5(2x - 3)(x + 2)$
9) $(x - 3)(x - y)$ 10) $(3x + 5)(3x - 5)$ 11) $4(x - 2)(x - 1)$ 12) $(4m - 9n)(4m + 9n)$
13) $y(2y - 3a)(2y + 3a)$ 14) $2(4x - 1)(x + 2)$ 15) $3(8x + 3)(x - 3)$

CHANGING SUBJECT OF A FORMULA

Ex J

1) $x = \frac{y+1}{7}$ 2) $x = 4y - 5$ 3) $x = 3(4y + 2)$ 4) $x = \frac{9y + 20}{12}$

Ex K

1) $t = \frac{32rP}{w}$ 2) $t = \pm\sqrt{\frac{32rP}{w}}$ 3) $t = \pm\sqrt{\frac{3V}{\pi h}}$ 4) $t = \frac{P^2 g}{2}$ 5) $t = v - \frac{Pag}{w}$ 6) $t = \pm\sqrt{\frac{r-a}{b}}$

Ex L

1) $x = \frac{c-3}{a-b}$ 2) $x = \frac{3a + 2k}{k - 3}$ 3) $x = \frac{2y+3}{5y-2}$ 4) $x = \frac{ab}{b-a}$

5) $x = \frac{4}{3y-5}$ 6) $x = \frac{3a^2 - b}{2}$ 7) $x = \frac{4}{3y^2 - 1}$ 8) $x = \sqrt[3]{\frac{w + 2y}{y - 3}}$

Completing the Square

Ex M

1 a) $(x + 2)^2 - 1$ b) $(x - 5)^2 - 28$ c) $(x - 4)^2 - 16$ d) $(x + 3)^2 - 9$ e) $(x - 1)^2 + 6$
2 a) $2(x - 2)^2 - 24$ b) $4(x - 1)^2 - 20$ c) $3(x + 2)^2 - 21$
3 a) $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$ b) $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$ 4. $9 - 2(3x + 1)^2$

Quadratic Equations

Ex. N

1) a) -1, -2 b) -1, 4 c) -5, 3 2) a) 0, -3 b) 0, 4 c) 2, -2 3) a) -3/2, -2 b) 1/2 , 5/2 c) 0.5, 2.5
4) a) $3 \pm\sqrt{13}$ b) $-4 \pm\sqrt{3}$ c) $6 \pm\sqrt{31}$ 5) a) -0.807, -6.19 b) 6.27, -1.27 c) 2.26, 0.736 6 1.23, 2.77