

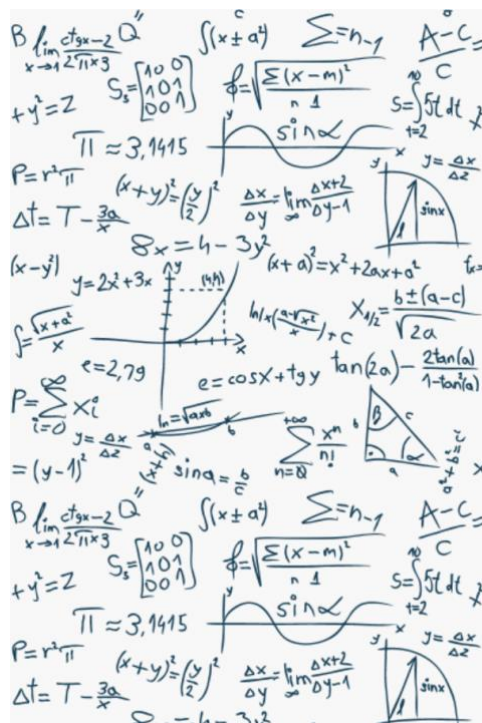


# MATHEMATICS

## Introduction to A level Mathematics

### Bridging Work

Summer 2025



## Getting Prepared for Maths A level

A level Mathematics contain a good deal of algebra and trigonometry, and it is important that you start your course confident in using all the techniques of algebraic manipulation that you learnt in your GCSE.

- Expanding brackets
- Solving linear equations
- Solving Simultaneous equations
- Factorising
- Rearranging formulae
- Completing the square
- Solving quadratic equations
- Trigonometry

These exercises have been collected together for you to practice these techniques. Write your solutions on lined paper. **Mark** and **correct** your work, where necessary using the answers at the end of the booklet using **green pen**.

Being successful in A levels does require a commitment to independent learning and we are asking you teach yourself the topic of inequalities, including using **set notation** and bring in notes on this topic as well as doing some practice questions.

## Preparing for UKMT Maths Challenges

For some of you, you may want to enter the **UKMT Senior Maths Challenge** which takes place on **Thursday 9<sup>th</sup> October**. All of you taking **Further Maths** will do this challenge and we recommend anyone wanting to study a maths related degree at a top University to participate. You will need to do some practice for this and past papers can be found here: [UKMT Past Papers](#)

For Girls, there is a new Mathematical Competition for Girls and a Mathematical Olympiad for Girls on the **Thursday 25<sup>th</sup> September**. [MOG Past papers](#)

# Bridging Task 1: Algebra Review

## EXPANDING BRACKETS

**Example 1** Expand  $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket
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**Example 2** Expand and simplify  $3(x + 5) - 4(2x + 3)$

$\begin{aligned} 3(x + 5) - 4(2x + 3) \\ = 3x + 15 - 8x - 12 \\ \\ = 3 - 5x \end{aligned}$	<p><b>1</b> Expand each set of brackets separately by multiplying <math>(x + 5)</math> by 3 and <math>(2x + 3)</math> by <math>-4</math></p> <p><b>2</b> Simplify by collecting like terms: <math>3x - 8x = -5x</math> and <math>15 - 12 = 3</math></p>
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**Example 3** Expand and simplify  $(x - 5)(2x + 3)$

$\begin{aligned} (x - 5)(2x + 3) \\ = x(2x + 3) - 5(2x + 3) \\ = 2x^2 + 3x - 10x - 15 \\ = 2x^2 - 7x - 15 \end{aligned}$	<p><b>1</b> Expand the brackets by multiplying <math>(2x + 3)</math> by <math>x</math> and <math>(2x + 3)</math> by <math>-5</math></p> <p><b>2</b> Simplify by collecting like terms: <math>3x - 10x = -7x</math></p>
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[Expanding Brackets 1](#)

[Expanding Brackets 2](#)

### EXERCISE A

Multiply out the following brackets and simplify.

1.  $7(4x + 5)$

9.  $(2x + 3y)(3x - 4y)$

2.  $-3(5x - 7)$

10.  $4(x - 2)(x + 3)$

3.  $5a - 4(3a - 1)$

11.  $(2y - 1)(2y + 1)$

4.  $4y + y(2 + 3y)$

12.  $(3 + 5x)(4 - x)$

5.  $-3x - (x + 4)$

13.  $(x + 2)(x + 3)(x + 4)$

6.  $5(2x - 1) - (3x - 4)$

14.  $(x - 3)^3$

7.  $(x + 2)(x + 3)$

15.  $(2x + 3)(3x - 5)(x - 7)$

8.  $(t - 5)(t - 2)$

16. The  $x^2$  term in the expansion of  $(3x + 4)(x^2 + px + 5)$  is  $-23x^2$   
Work out the value of  $p$ .

## Two Special Cases

### Perfect Square:

$$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$$
$$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$$

### Difference of two squares:

$$(x - a)(x + a) = x^2 - a^2$$
$$(x - 3)(x + 3) = x^2 - 3^2$$
$$= x^2 - 9$$

### EXERCISE B Multiply out

1.  $(x - 1)^2$

2.  $(3x + 5)^2$

3.  $(7x - 2)^2$

4.  $(x + 2)(x - 2)$

5.  $(3x + 1)(3x - 1)$

6.  $(5y - 3)(5y + 3)$

## LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in  $x$ . A linear equation does not contain any  $x^2$  or  $x^3$  terms.

**Example 1:** Solve the equation  $64 - 3x = 25$

**Solution:** There are various ways to solve this equation. One approach is as follows:

Step 1: Add  $3x$  to both sides (so that the  $x$  term is positive):  $64 = 3x + 25$

Step 2: Subtract 25 from both sides:  $39 = 3x$

Step 3: Divide both sides by 3:  $13 = x$

So the solution is  $x = 13$ .

**Example 2:** Solve the equation  $6x + 7 = 5 - 2x$ .

**Solution:**

Step 1: Begin by adding  $2x$  to both sides  $8x + 7 = 5$   
(to ensure that the  $x$  terms are together on the same side)

Step 2: Subtract 7 from each side:  $8x = -2$

Step 3: Divide each side by 8:  $x = -\frac{1}{4}$

**Example 3:** Solve the equation  $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets:  $6x - 4 = 20 - 3x - 6$   
(taking care of the negative signs)

Step 2: Simplify the right hand side:  $6x - 4 = 14 - 3x$

Step 3: Add  $3x$  to each side:  $9x - 4 = 14$

Step 4: Add 4:  $9x = 18$

Step 5: Divide by 9:  $x = 2$

**Exercise C:** Solve the following equations, showing each step in your working:

1)  $2x + 5 = 19$

2)  $5x - 2 = 13$

3)  $11 - 4x = 5$

4)  $5 - 7x = -9$

5)  $11 + 3x = 8 - 2x$

6)  $7x + 2 = 4x - 5$

**Exercise D:** Solve the following equations.

1)  $5(2x - 4) = 4$

2)  $4(2 - x) = 3(x - 9)$

3)  $8 - (x + 3) = 4$

4)  $14 - 3(2x + 3) = 2$

## EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

**Example 4:** Solve the equation  $\frac{y}{2} + 5 = 11$

**Solution:**

Step 1: Multiply through by 2 (the denominator in the fraction):  $y + 10 = 22$

Step 2: Subtract 10:  $y = 12$

**Example 5:** Solve the equation  $\frac{1}{3}(2x + 1) = 5$

**Solution:**

Step 1: Multiply by 3 (to remove the fraction)  $2x + 1 = 15$

Step 2: Subtract 1 from each side  $2x = 14$

Step 3: Divide by 2  $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

[Linear Equations 1](#)

[Linear Equations 2](#)

**Example 6:** Solve the equation  $\frac{x+1}{4} + \frac{x+2}{5} = 2$

**Solution:**

Step 1: Find the lowest common denominator:

The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator

$$\frac{20(x+1)}{4} + \frac{20(x+2)}{5} = 40$$

Step 3: Simplify the left hand side:

$$\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}} + \frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}} = 40$$

$$5(x+1) + 4(x+2) = 40$$

Step 4: Multiply out the brackets:

$$5x + 5 + 4x + 8 = 40$$

Step 5: Simplify the equation:

$$9x + 13 = 40$$

Step 6: Subtract 13

$$9x = 27$$

Step 7: Divide by 9:

$$x = 3$$

**Example 7:** Solve the equation  $x + \frac{x-2}{4} = 2 - \frac{3-5x}{6}$

**Solution:** The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x + \frac{12(x-2)}{4} = 24 - \frac{12(3-5x)}{6}$$

Simplify

$$12x + 3(x-2) = 24 - 2(3-5x)$$

Expand brackets

$$12x + 3x - 6 = 24 - 6 + 10x$$

Simplify

$$15x - 6 = 18 + 10x$$

Subtract 10x

$$5x - 6 = 18$$

Add 6

$$5x = 24$$

Divide by 5

$$x = 4.8$$

**Exercise E:** Solve these equations

1)  $\frac{1}{2}(x+3) = 5$

2)  $\frac{2x}{3} - 1 = \frac{x}{3} + 4$

3)  $\frac{y}{4} + 3 = 5 - \frac{y}{3}$

4)  $\frac{x-2}{7} = 2 + \frac{3-x}{14}$

5)  $\frac{7x-1}{2} = 13 - x$

6)  $\frac{y-1}{2} + \frac{y+1}{3} = \frac{2y+5}{6}$

7)  $2x + \frac{x-1}{2} = \frac{5x+3}{3}$

## FORMING EQUATIONS

**Example 8:** Find three consecutive numbers so that their sum is 96.

**Solution:** Let the first number be  $n$ , then the second is  $n + 1$  and the third is  $n + 2$ .

Therefore  $n + (n + 1) + (n + 2) = 96$

$$3n + 3 = 96$$

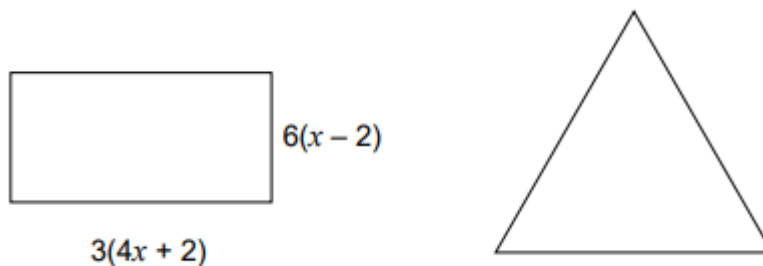
$$3n = 93$$

$$n = 31$$

So the numbers are 31, 32 and 33.

### Exercise F:

- 1) Find 3 consecutive even numbers so that their sum is 108.
- 2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.
- 3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has. Form an equation, letting  $n$  be the number of photographs one girl had at the **beginning**. Hence find how many each has **now**.
- 4) The rectangle and the equilateral triangle have equal perimeters.



Work out an expression, in terms of  $x$ , for the length of a side of the triangle.  
Give your answer in its simplest form.

- 5) (a) Jean pays for gas, electricity and water every month. Her gas bill is £ $G$  per month. Her electricity bill is £10 per month more than her gas bill. Each month her water bill is half her electricity bill. Write down an expression for the total cost of the bills for 1 year.
- (b) Jean pays a total of £1830 a year for her gas, electricity and water bills. Use your equation to work out how much Jean pays for electricity for one month.

# SIMULTANEOUS EQUATIONS

An example of a pair of simultaneous equations is  $3x + 2y = 8$  (1)  
 $5x + y = 11$  (2)

In these equations,  $x$  and  $y$  stand for two numbers. We can solve these equations in order to find the values of  $x$  and  $y$  by eliminating one of the letters from the equations.

In these equations it is simplest to eliminate  $y$ . We do this by making the coefficients of  $y$  the same in both equations. This can be achieved by multiplying equation (2) by 2, so that both equations contain  $2y$ :

$$\begin{array}{rcl} 3x + 2y & = & 8 \quad (1) \\ 10x + 2y & = & 22 \quad 2 \times (2) = (3) \end{array}$$

To eliminate the  $y$  terms, we subtract equation (3) from equation (1). We get:  $7x = 14$   
i.e.  $x = 2$

To find  $y$ , we substitute  $x = 2$  into one of the original equations. For example if we put it into (2):

$$\begin{array}{rcl} 10 + y & = & 11 \\ y & = & 1 \end{array}$$

Therefore the solution is  $x = 2, y = 1$ .

**Example:** Solve  $2x + 5y = 16$  (1)  
 $3x - 4y = 1$  (2)

**Solution:** We begin by getting the same number of  $x$  or  $y$  appearing in both equation. We can get  $20y$  in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$$\begin{array}{rcl} 8x + 20y & = & 64 \quad (3) \\ 15x - 20y & = & 5 \quad (4) \end{array}$$

As the SIGNS in front of  $20y$  are DIFFERENT, we can eliminate the  $y$  terms from the equations by ADDING:

$$\begin{array}{rcl} 23x & = & 69 \quad (3) + (4) \\ \text{i.e. } x & = & 3 \end{array}$$

Substituting this into equation (1) gives:

$$\begin{array}{rcl} 6 + 5y & = & 16 \\ 5y & = & 10 \end{array}$$

So...  $y = 2$

The solution is  $x = 3, y = 2$ .

## Linear Simultaneous Equations 1

## Linear Simultaneous Equations 2

**Exercise G:** Solve the pairs of simultaneous equations in the following questions:

1)  $x + 2y = 7$   
 $3x + 2y = 9$

2)  $x + 3y = 0$   
 $3x + 2y = -7$

3)  $3x - 2y = 4$   
 $2x + 3y = -6$

4)  $9x - 2y = 25$   
 $4x - 5y = 7$

5)  $4a + 3b = 22$   
 $5a - 4b = 43$

6)  $3p + 3q = 15$   
 $2p + 5q = 14$



# FACTORISING

## Common factors

We can factorise some expressions by taking out a common factor.

**Example 1:** Factorise  $12x - 30$

**Solution:** 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:  
$$12x - 30 = 6(2x - 5)$$

**Example 2:** Factorise  $6x^2 - 2xy$

**Solution:** 2 is a common factor to both 6 and 2. Both terms also contain an  $x$ .  
So we factorise by taking  $2x$  outside a bracket.  
$$6x^2 - 2xy = 2x(3x - y)$$

**Example 3:** Factorise  $9x^3y^2 - 18x^2y$

**Solution:** 9 is a common factor to both 9 and 18.  
The highest power of  $x$  that is present in both expressions is  $x^2$ .  
There is also a  $y$  present in both parts.  
So we factorise by taking  $9x^2y$  outside a bracket:  
$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

**Example 4:** Factorise  $3x(2x - 1) - 4(2x - 1)$

**Solution:** There is a common bracket as a factor.  
So we factorise by taking  $(2x - 1)$  out as a factor.  
The expression factorises to  $(2x - 1)(3x - 4)$

## Exercise H

Factorise each of the following

1)  $3x + xy$

2)  $4x^2 - 2xy$

3)  $pq^2 - p^2q$

4)  $3pq - 9q^2$

5)  $2x^3 - 6x^2$

6)  $8a^5b^2 - 12a^3b^4$

7)  $5y(y - 1) + 3(y - 1)$

## Factorising quadratics

### Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$

The method is:

Step 1: Form two brackets  $(x \dots)(x \dots)$

Step 2: Find two numbers that multiply to give  $c$  and add to make  $b$ . These two numbers get written at the other end of the brackets.

**Example 1:** Factorise  $x^2 - 9x - 10$ .

**Solution:** We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.

Therefore  $x^2 - 9x - 10 = (x - 10)(x + 1)$ .

### General quadratics: Factorising quadratics of the form $ax^2 + bx + c$

The method is:

Step 1: Find two numbers that multiply together to make  $ac$  and add to make  $b$ .

Step 2: Split up the  $bx$  term using the numbers found in step 1.

Step 3: Factorise the front and back pair of expressions as fully as possible.

Step 4: There should be a common bracket. Take this out as a common factor.

**Example 2:** Factorise  $6x^2 + x - 12$ .

**Solution:** We need to find two numbers that multiply to make  $6 \times -12 = -72$  and add to make 1. These two numbers are -8 and 9.

Therefore, 
$$\begin{aligned} 6x^2 + x - 12 &= 6x^2 - 8x + 9x - 12 \\ &= 2x(3x - 4) + 3(3x - 4) && \text{(the two brackets must be identical)} \\ &= (3x - 4)(2x + 3) \end{aligned}$$

### Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that  $x^2 - a^2 = (x + a)(x - a)$ .

Therefore:  $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

$$16x^2 - 25 = (4x)^2 - 5^2 = (2x + 5)(2x - 5)$$

Also notice that:  $2x^2 - 8 = 2(x^2 - 4) = 2(x + 4)(x - 4)$

and  $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x + 4y)(x - 4y)$

### Factorising by pairing

We can factorise expressions like  $2x^2 + xy - 2x - y$  using the method of factorising by pairing:

$$\begin{aligned} 2x^2 + xy - 2x - y &= x(2x + y) - 1(2x + y) && \text{(factorise front and back pairs, ensuring both} \\ & && \text{brackets are identical)} \\ &= (2x + y)(x - 1) \end{aligned}$$

**Example** Simplify  $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

For the numerator:

$$\begin{aligned}x^2 - 4x - 21 &= x^2 - 7x + 3x - 21 \\&= x(x - 7) + 3(x - 7) \\&= (x - 7)(x + 3)\end{aligned}$$

$$\begin{aligned}\text{So } \frac{x^2 - 4x - 21}{2x^2 + 9x + 9} &= \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)} \\&= \frac{x - 7}{2x + 3}\end{aligned}$$

For the denominator:

$$\begin{aligned}2x^2 + 9x + 9 &= 2x^2 + 6x + 3x + 9 \\&= 2x(x + 3) + 3(x + 3) \\&= (x + 3)(2x + 3)\end{aligned}$$

### Factorising 1

### Factorising 2

### Factorising 3

#### Exercise I

Factorise;

- |                     |                                  |                                    |
|---------------------|----------------------------------|------------------------------------|
| 1) $x^2 - x - 6$    | 2) $x^2 + 6x - 16$               | 3) $2x^2 + 5x + 2$                 |
| 4) $2x^2 - 3x$      | 5) $3x^2 + 5x - 2$               | 6) $2y^2 + 17y + 21$               |
| 7) $7y^2 - 10y + 3$ | 8) $10x^2 + 5x - 30$             | 9) $x^2 - 3x - xy + 3y^2$          |
| 10) $9x^2 - 25$     | 11) $4x^2 - 12x + 8$             | 12) $16m^2 - 81n^2$                |
| 13) $4y^3 - 9a^2y$  | 14) $8(x + 1)^2 - 2(x + 1) - 10$ | 15) $6(2x - 3)^2 + 3(3x + 4) - 93$ |

16) Simplify the algebraic fractions.

a  $\frac{2x^2 + 4x}{x^2 - x}$

b  $\frac{x^2 + 3x}{x^2 + 2x - 3}$

c  $\frac{x^2 - 2x - 8}{x^2 - 4x}$

d  $\frac{x^2 - 5x}{x^2 - 25}$

e  $\frac{x^2 - x - 12}{x^2 - 4x}$

17) Simplify

a  $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

b  $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

c  $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

d  $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

e  $\frac{(x + 2)^2 + 3(x + 2)^2}{x^2 - 4}$

# REARRANGING FORMULA

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

**Example 1:** Make  $x$  the subject of the formula  $y = 4x + 3$ .

**Solution:**  
Subtract 3 from both sides:  $y = 4x + 3$   
 $y - 3 = 4x$   
Divide both sides by 4;  $\frac{y-3}{4} = x$   
So  $x = \frac{y-3}{4}$  is the same equation but with  $x$  the subject.

**Example 2:** Make  $x$  the subject of  $y = 2 - 5x$

**Solution:** Notice that in this formula the  $x$  term is negative.  
 $y = 2 - 5x$   
Add  $5x$  to both sides  $y + 5x = 2$  (the  $x$  term is now positive)  
Subtract  $y$  from both sides  $5x = 2 - y$   
Divide both sides by 5  $x = \frac{2-y}{5}$

**Example 3:** The formula  $C = \frac{5(F-32)}{9}$  is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make  $F$  the subject.

$C = \frac{5(F-32)}{9}$   
Multiply by 9  $9C = 5(F-32)$  (this removes the fraction)  
Expand the brackets  $9C = 5F - 160$   
Add 160 to both sides  $9C + 160 = 5F$   
Divide both sides by 5  $\frac{9C + 160}{5} = F$   
Therefore the required rearrangement is  $F = \frac{9C + 160}{5}$ .

## Rearranging Formula 1

### Exercise J

Make  $x$  the subject of each of these formulae:

1)  $y = 7x - 1$

2)  $y = \frac{x+5}{4}$

3)  $4y = \frac{x}{3} - 2$

4)  $y = \frac{4(3x-5)}{9}$

## Rearranging equations involving squares and square roots

**Example 4:** Make  $x$  the subject of  $x^2 + y^2 = w^2$

**Solution:**

$$x^2 + y^2 = w^2$$

Subtract  $y^2$  from both sides:

$$x^2 = w^2 - y^2 \quad (\text{this isolates the term involving } x)$$

Square root both sides:

$$x = \pm\sqrt{w^2 - y^2}$$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

**Example 5:** Make  $a$  the subject of the formula  $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

**Solution:**

$$t = \frac{1}{4}\sqrt{\frac{5a}{h}}$$

Multiply by 4

$$4t = \sqrt{\frac{5a}{h}}$$

Square both sides

$$16t^2 = \frac{5a}{h}$$

Multiply by  $h$ :

$$16t^2h = 5a$$

Divide by 5:

$$\frac{16t^2h}{5} = a$$

### Exercise K

Make  $t$  the subject of each of the following

1)  $P = \frac{wt}{32r}$

2)  $P = \frac{wt^2}{32r}$

3)  $V = \frac{1}{3}\pi t^2h$

4)  $P = \sqrt{\frac{2t}{g}}$

5)  $Pa = \frac{w(v-t)}{g}$

6)  $r = a + bt^2$

## More difficult examples

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

**Example 6:** Make  $t$  the subject of the formula  $a - xt = b + yt$

**Solution:**

$$a - xt = b + yt$$

Start by collecting all the  $t$  terms on the right hand side:

Add  $xt$  to both sides: 
$$a = b + yt + xt$$

Now put the terms without a  $t$  on the left hand side:

Subtract  $b$  from both sides: 
$$a - b = yt + xt$$

Factorise the RHS: 
$$a - b = t(y + x)$$

Divide by  $(y + x)$ : 
$$\frac{a - b}{y + x} = t$$

So the required equation is 
$$t = \frac{a - b}{y + x}$$

**Example 7:** Make  $W$  the subject of the formula  $T - W = \frac{Wa}{2b}$

**Solution:** This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by  $2b$ : 
$$2bT - 2bW = Wa$$

Add  $2bW$  to both sides: 
$$2bT = Wa + 2bW \quad (\text{this collects the } W\text{'s together})$$

Factorise the RHS: 
$$2bT = W(a + 2b)$$

Divide both sides by  $a + 2b$ : 
$$W = \frac{2bT}{a + 2b}$$

## Rearranging Formula 2

### Exercise L

Make  $x$  the subject of these formulae:

1)  $ax + 3 = bx + c$

2)  $3(x + a) = k(x - 2)$

3)  $y = \frac{2x + 3}{5x - 2}$

4)  $\frac{x}{a} = 1 + \frac{x}{b}$

5)  $xy = \frac{5x + 4}{3}$

6)  $a = \sqrt{\frac{b + 2x}{3}}$

7)  $y = \frac{\sqrt{x + 4}}{3x}$

8)  $y = \frac{3x^3 + w}{x^3 - 2}$

# Completing the square

## Key points

- Completing the square for a quadratic rearranges  $ax^2 + bx + c$  into the form  $p(x + q)^2 + r$
- If  $a \neq 1$ , then factorise using  $a$  as a common factor.

**Example 1** Complete the square for the quadratic expression  $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	<p><b>1</b> Write <math>x^2 + bx + c</math> in the form <math>\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c</math></p> <p><b>2</b> Simplify</p>
---	---

**Example 2** Write  $2x^2 - 5x + 1$  in the form  $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	<p><b>1</b> Before completing the square write <math>ax^2 + bx + c</math> in the form <math>a\left(x^2 + \frac{b}{a}x\right) + c</math></p> <p><b>2</b> Now complete the square by writing <math>x^2 - \frac{5}{2}x</math> in the form <math>\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2</math></p> <p><b>3</b> Expand the square brackets – don't forget to multiply <math>\left(\frac{5}{4}\right)^2</math> by the factor of 2</p> <p><b>4</b> Simplify</p>
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[Completing the Square 1](#)

[Completing the Square 2](#)

[Completing the Square 3](#)

[Completing the Square 4](#)

## Exercise M

- 1 Write the following quadratic expressions in the form  $(x + p)^2 + q$
- a**  $x^2 + 4x + 3$                                       **b**  $x^2 - 10x - 3$   
**c**  $x^2 - 8x$     **d**  $x^2 + 6x$   
**e**  $x^2 - 2x + 7$
- 2 Write the following quadratic expressions in the form  $p(x + q)^2 + r$
- a**  $2x^2 - 8x - 16$                                       **b**  $4x^2 - 8x - 16$   
**c**  $3x^2 + 12x - 9$
- 3 Complete the square.
- a**  $2x^2 + 3x + 6$                                       **b**  $3x^2 - 2x$   
**c**  $5x^2 + 3x$     **d**  $3x^2 + 5x + 3$
- 4 Write  $(25x^2 + 30x + 12)$  in the form  $(ax + b)^2 + c$ .
- 5 Write  $7 - 12x - 18x^2$  in the form  $a - 2(bx + c)^2$

## SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form  $ax^2 + bx + c = 0$ .

There are three methods that are commonly used for solving quadratic equations:

\* factorising      \* the quadratic formula      \* completing the square

### Method : Factorising

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of  $x^2$  is positive.

**Example :** Solve  $x^2 - 3x + 2 = 0$

Factorise                       $(x - 1)(x - 2) = 0$

Either  $(x - 1) = 0$  or  $(x - 2) = 0$

So the solutions are  $x = 1$  or  $x = 2$

Note: The individual values  $x = 1$  and  $x = 2$  are called the **roots** of the equation.

### Method: Completing the Square

**Example :** Solve  $x^2 - 2x - 1 = 0$

Write in form  $(x - p)^2 + q = 0$                        $(x - 1)^2 - 2 = 0$

Rearrange to make  $x$  the subject:               $(x - 1)^2 = 2$   
    $x - 1 = \pm\sqrt{2}$

So  $x = 1 \pm\sqrt{2}$



## Method: Quadratic Formula

**Example :** Solve  $3x^2 - 8x - 7 = 0$   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Write down values of  $a, b, c$   $a = 3, b = -8, c = -7$

Substitute into formula:  $x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$

Simplify:  $x = \frac{8 \pm \sqrt{148}}{6}$

Evaluate:  $x = 3.36, -1.53$  (3.s.f.)

## Solving Quadratic Equations

### Using Quadratic Formula

#### Exercise N

1) Use factorisation to solve the following equations:

a)  $x^2 + 3x + 2 = 0$

b)  $x^2 - 3x - 4 = 0$

c)  $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a)  $x^2 + 3x = 0$

b)  $x^2 - 4x = 0$

c)  $4 - x^2 = 0$

3) Solve the following equations by factorising.

a)  $2x^2 + 7x + 6 = 0$

b)  $8x^2 - 24x + 10 = 0$

c)  $6x^2 - 5x - 4 = 0$

4) Solve the following equations by completing the square. Give your answers in surd form.

a)  $x^2 - 6x - 4 = 0$

b)  $x^2 + 8x + 13 = 0$

c)  $2x^2 - 24x + 10 = 0$

5) Solve the following equations by quadratic formula. Round your answers to 3 significant figures.

a)  $x^2 + 7x + 5 = 0$

b)  $x^2 - 5x - 8 = 0$

c)  $3x^2 + 5 = 9x$

6) Solve by completing the square.

a)  $5x^2 + 3x - 4 = 0$

b)  $(x - 4)(x + 2) = 5$

c)  $2x^2 + 6x - 7 = 0$

d)  $x^2 - 5x + 3 = 0$

7) Solve  $\frac{3}{x-2} + \frac{2}{x-1} = 5$  Do not use trial and improvement. Write your solutions to 3 sig. figs.

## Task 2: Trigonometry Review

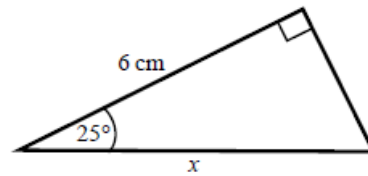
### Trigonometry Review

## TRIGONOMETRY IN RIGHT-ANGLED TRIANGLES

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle  $\theta$  is called the opposite
  - the side next to the angle  $\theta$  is called the adjacent.
- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle  $\theta$ ,  $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle  $\theta$ ,  $\cos \theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle  $\theta$ ,  $\tan \theta = \frac{\text{opp}}{\text{adj}}$
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions:  $\sin^{-1}$ ,  $\cos^{-1}$ ,  $\tan^{-1}$ .
- The sine, cosine and tangent of some angles may be written exactly.

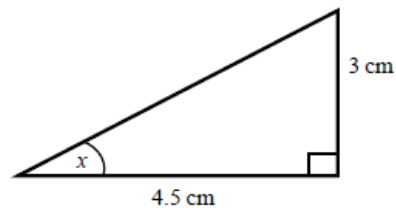
	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

**Example 1** Calculate the length of side  $x$ .  
Give your answer correct to 3 significant figures.



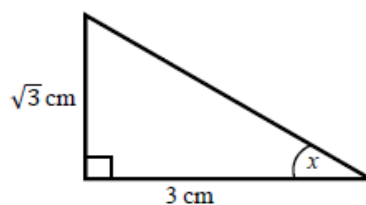
$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\cos 25^\circ = \frac{6}{x}$ $x = \frac{6}{\cos 25^\circ}$ $x = 6.620\,267\,5\dots$ $x = 6.62\text{ cm}$	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the adjacent and the hypotenuse so use the cosine ratio.</li> <li>3 Substitute the sides and angle into the cosine ratio.</li> <li>4 Rearrange to make <math>x</math> the subject.</li> <li>5 Use your calculator to work out <math>6 \div \cos 25^\circ</math>.</li> <li>6 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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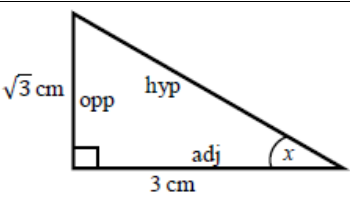
**Example 2** Calculate the size of angle  $x$ .  
Give your answer correct to 3 significant figures.



$\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{3}{4.5}$ $x = \tan^{-1} \left( \frac{3}{4.5} \right)$ $x = 33.690\,067\,5\dots$ $x = 33.7^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the opposite and the adjacent so use the tangent ratio.</li> <li>3 Substitute the sides and angle into the tangent ratio.</li> <li>4 Use <math>\tan^{-1}</math> to find the angle.</li> <li>5 Use your calculator to work out <math>\tan^{-1}(3 \div 4.5)</math>.</li> <li>6 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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**Example 3** Calculate the exact size of angle  $x$ .

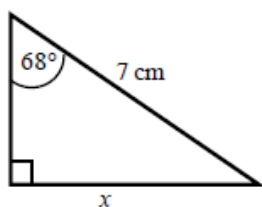


 $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the opposite and the adjacent so use the tangent ratio.</li> <li>3 Substitute the sides and angle into the tangent ratio.</li> <li>4 Use the table from the key points to find the angle.</li> </ol>
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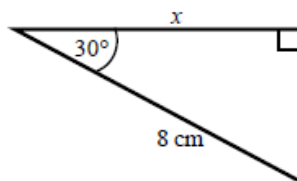
## Exercise A

- 1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

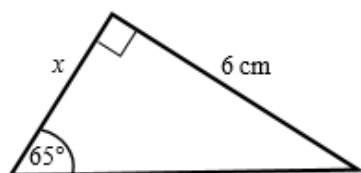
a



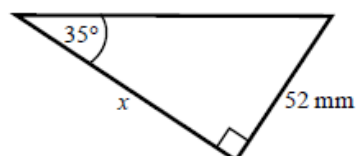
b



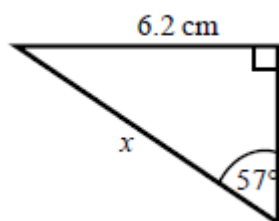
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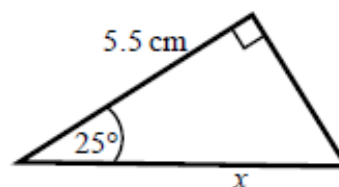
d



e

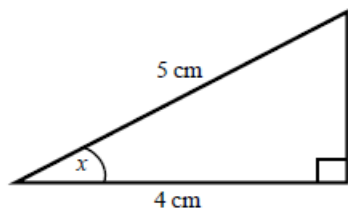


f

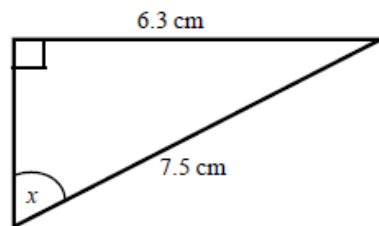


- 2 Calculate the size of angle  $x$  in each triangle.  
Give your answers correct to 1 decimal place.

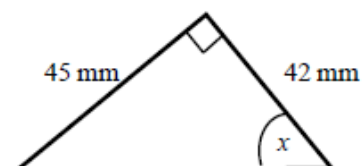
a



b



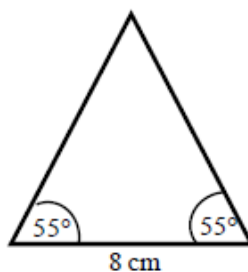
c



- 3 Work out the height of the isosceles triangle.  
Give your answer correct to 3 significant figures.

**Hint:**

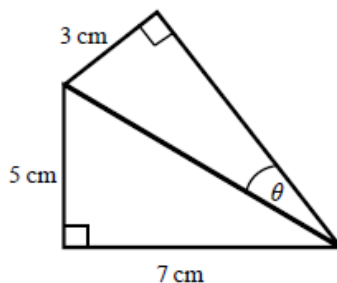
Split the triangle into two right-angled triangles.



- 4 Calculate the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.

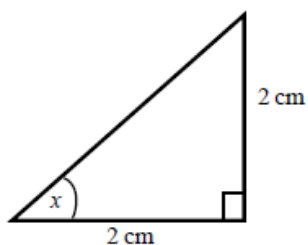
**Hint:**

First work out the length of the common side to both triangles, leaving your answer in surd form.

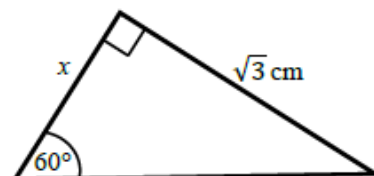


- 5 Find the exact value of  $x$  in each triangle.

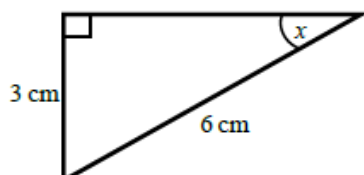
a



b



c



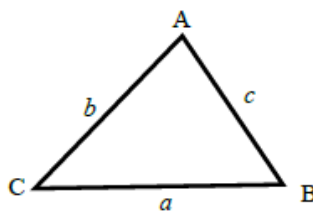
# TRIGONOMETRY IN ANY TRIANGLE

## THE COSINE RULE

$a$  is the side opposite angle  $A$ .

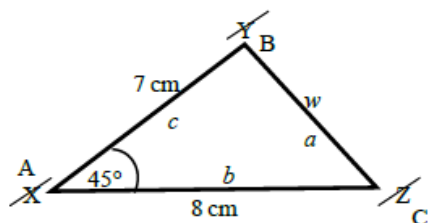
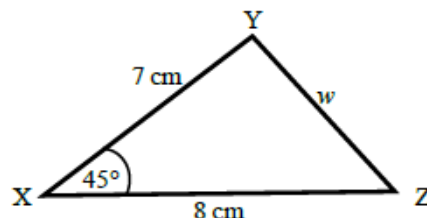
$b$  is the side opposite angle  $B$ .

$c$  is the side opposite angle  $C$ .



- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula  $a^2 = b^2 + c^2 - 2bc \cos A$ .
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ .

**Example 4** Work out the length of side  $w$ .  
Give your answer correct to 3 significant figures.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45^\circ$$

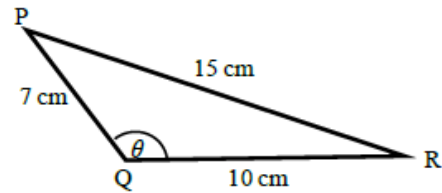
$$w^2 = 33.804\,040\,51\dots$$

$$w = \sqrt{33.804\,040\,51}$$

$$w = 5.81 \text{ cm}$$

- Always start by labelling the angles and sides.
- Write the cosine rule to find the side.
- Substitute the values  $a$ ,  $b$  and  $A$  into the formula.
- Use a calculator to find  $w^2$  and then  $w$ .
- Round your final answer to 3 significant figures and write the units in your answer.

**Example 5** Work out the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.

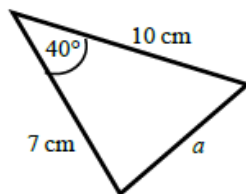


$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$ $\cos \theta = \frac{-76}{140}$ $\theta = 122.878\ 349\dots$ $\theta = 122.9^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the cosine rule to find the angle.</li> <li>3 Substitute the values <math>a</math>, <math>b</math> and <math>c</math> into the formula.</li> <li>4 Use <math>\cos^{-1}</math> to find the angle.</li> <li>5 Use your calculator to work out <math>\cos^{-1}(-76 \div 140)</math>.</li> <li>6 Round your answer to 1 decimal place and write the units in your answer.</li> </ol>
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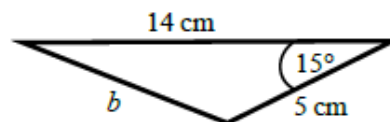
## Exercise B

**1** Work out the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.

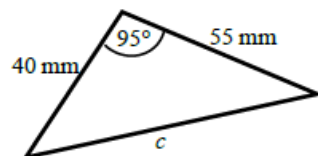
**a**



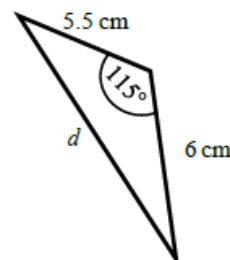
**b**



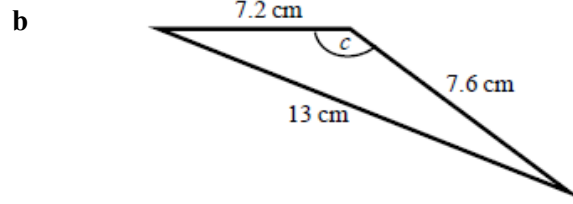
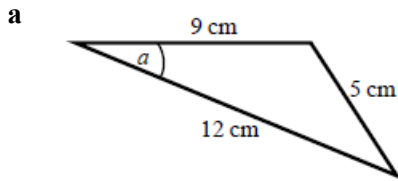
**c**



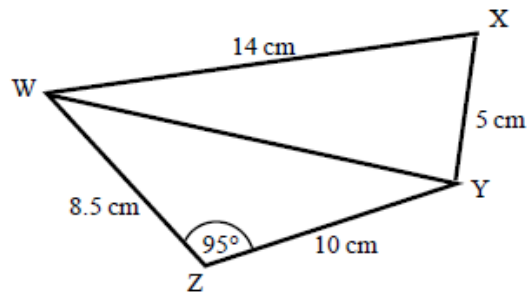
**d**



- 2 Calculate the angles labelled  $\theta$  in each triangle.  
Give your answer correct to 1 decimal place.

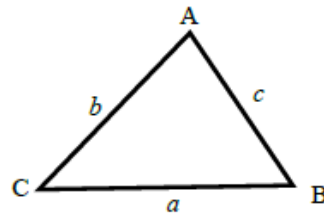


- 3 a Work out the length of WY.  
Give your answer correct to 3 significant figures.
- b Work out the size of angle WXY.  
Give your answer correct to 1 decimal place.



## THE SINE RULE

- $a$  is the side opposite angle A.
- $b$  is the side opposite angle B.
- $c$  is the side opposite angle C.

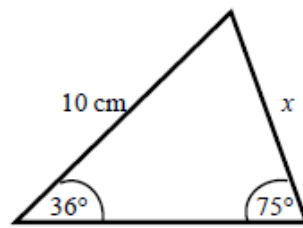


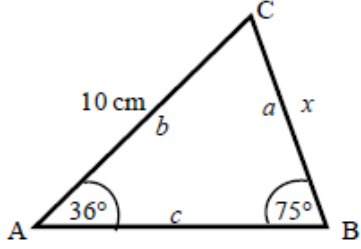
- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ .
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .



**Example 6**

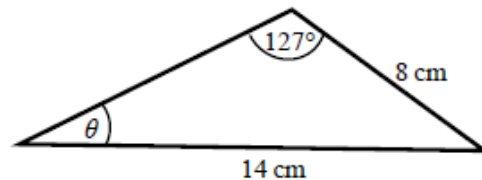
Work out the length of side  $x$ .  
Give your answer correct to 3 significant figures.

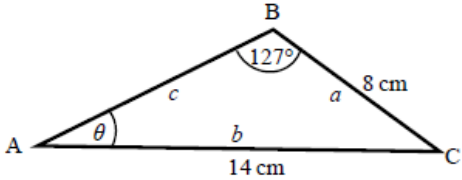


 $\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 36^\circ} = \frac{10}{\sin 75^\circ}$ $x = \frac{10 \times \sin 36^\circ}{\sin 75^\circ}$ $x = 6.09 \text{ cm}$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the sine rule to find the side.</li> <li>3 Substitute the values <math>a</math>, <math>b</math>, <math>A</math> and <math>B</math> into the formula.</li> <li>4 Rearrange to make <math>x</math> the subject.</li> <li>5 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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**Example 7**

Work out the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.

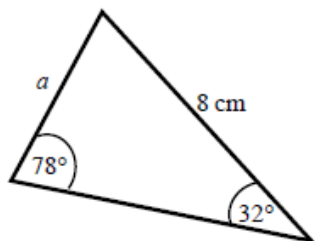


 $\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{8} = \frac{\sin 127^\circ}{14}$ $\sin \theta = \frac{8 \times \sin 127^\circ}{14}$ $\theta = 27.2^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the sine rule to find the angle.</li> <li>3 Substitute the values <math>a</math>, <math>b</math>, <math>A</math> and <math>B</math> into the formula.</li> <li>4 Rearrange to make <math>\sin \theta</math> the subject.</li> <li>5 Use <math>\sin^{-1}</math> to find the angle. Round your answer to 1 decimal place and write the units in your answer.</li> </ol>
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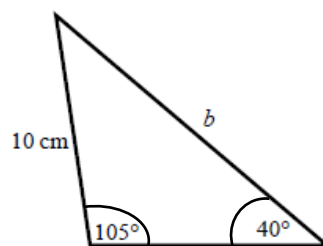
## Exercise C

- 1 Find the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.

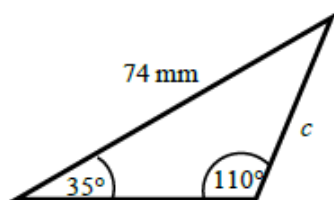
a



b

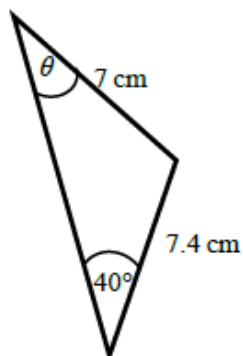


c

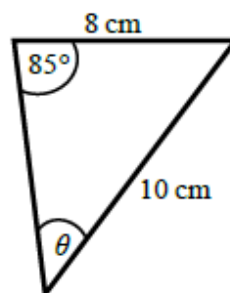


- 2 Calculate the angles labelled  $\theta$  in each triangle.  
Give your answer correct to 1 decimal place.

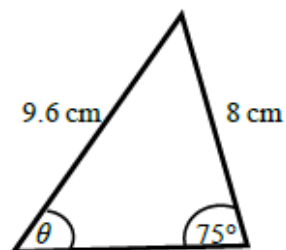
a



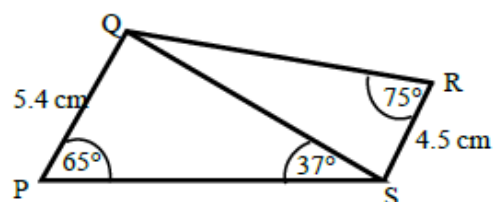
b



c

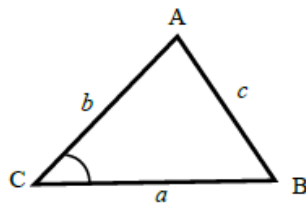


- 3 a Work out the length of QS.  
Give your answer correct to 3 significant figures.
- b Work out the size of angle RQS.  
Give your answer correct to 1 decimal place.

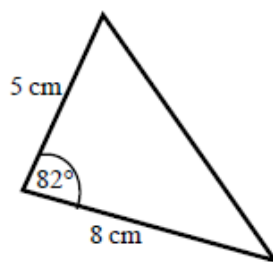


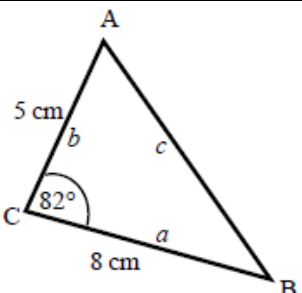
# AREA OF ANY TRIANGLE

- $a$  is the side opposite angle A.  
 $b$  is the side opposite angle B.  
 $c$  is the side opposite angle C.
- The area of the triangle is  $\frac{1}{2}ab \sin C$ .



**Example 8** Find the area of the triangle.

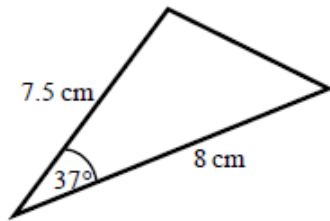


 <p>Area = <math>\frac{1}{2}ab \sin C</math></p> <p>Area = <math>\frac{1}{2} \times 8 \times 5 \times \sin 82^\circ</math></p> <p>Area = 19.805 361...</p> <p>Area = 19.8 cm<sup>2</sup></p>	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides and angles of the triangle.</li> <li>2 State the formula for the area of a triangle.</li> <li>3 Substitute the values of <math>a</math>, <math>b</math> and <math>C</math> into the formula for the area of a triangle.</li> <li>4 Use a calculator to find the area.</li> <li>5 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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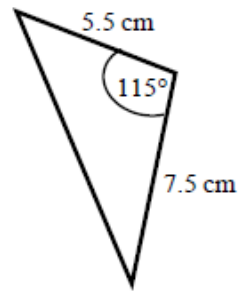
## Exercise D

- 1 Work out the area of each triangle.  
Give your answers correct to 3 significant figures.

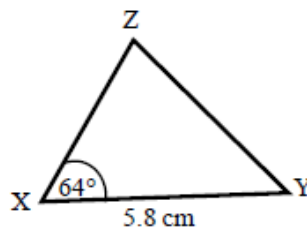
a



b



- 2 The area of triangle XYZ is 13.3 cm<sup>2</sup>.  
Work out the length of XZ.

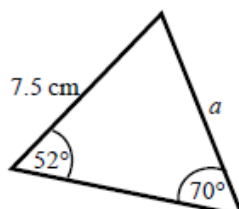


## Mixed

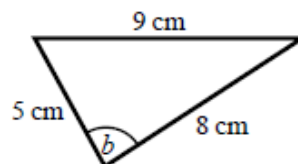
### Exercise E

- 1 Find the size of each lettered angle or side.  
Give your answers correct to 3 significant figures.

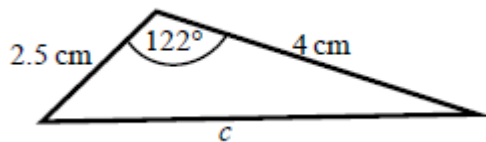
a



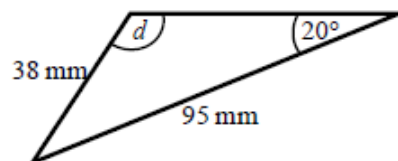
b



c



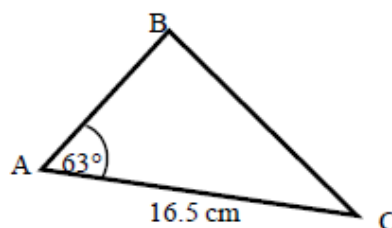
d



#### Hint:

For each one, decide whether to use the cosine or sine rule.

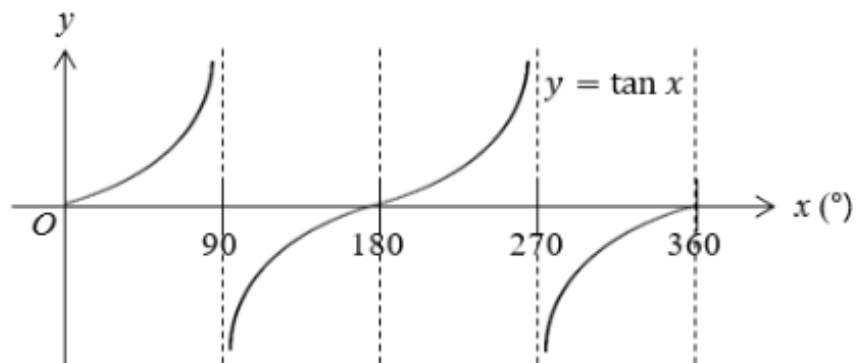
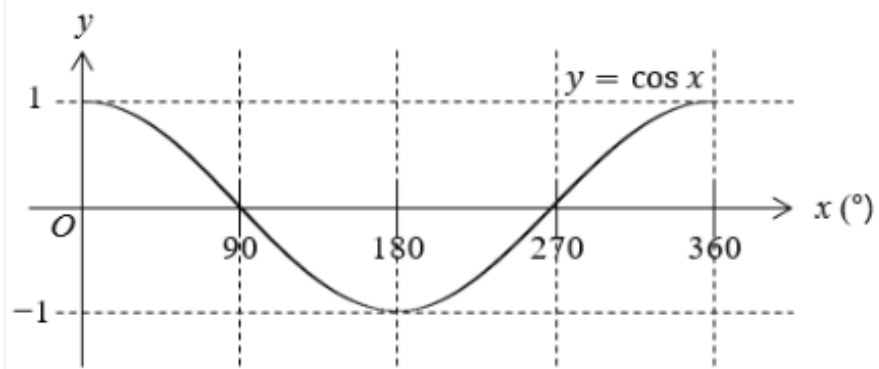
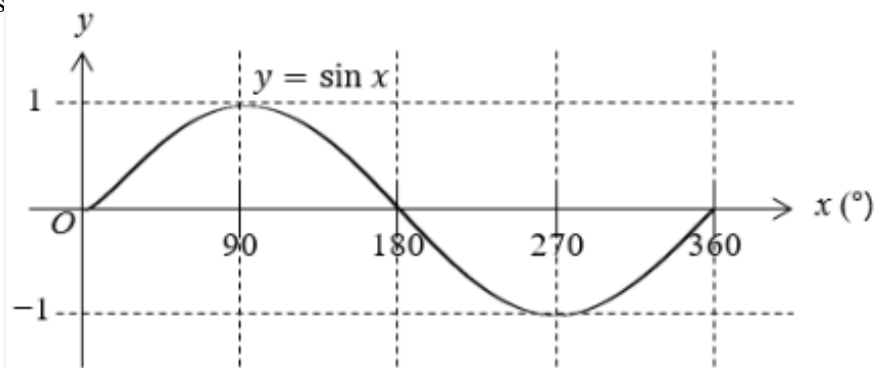
- 2 The area of triangle ABC is 86.7 cm<sup>2</sup>.  
Work out the length of BC.  
Give your answer correct to 3 significant figures.



# TRIGONOMETRICAL GRAPHS

Learn these shapes and be able to s

- $y = \sin x$
- $y = \cos x$
- $y = \tan x$



## Task 3: Set Notation and Inequalities

### 1: Watch these 3 videos on set notation and write notes

[Introduction to Inequalities and Set Notation and Interval Notation](#)

[Writing Inequalities in Set Notation and Interval Notation](#)

[Write answers to Linear Inequalities in Set Notation](#)

### Task 2: Solve these problems

If you need notes, these are more videos:

[Intro to Linear Inequalities](#)

[Solving Linear Inequalities](#)

[Solve Complex Linear Inequalities](#)

[Quadratic Inequalities Example](#)

[More Quadratic Inequalities Examples](#)

[Solve Complex Quadratic Inequalities](#)

## Inequalities

### 1. Solve these linear inequalities

- a)  $4x + 21 < x - 18$     b)  $4 - 6x \geq 19$     c)  $6(y + 3) \leq 2(5y - 9)$   
d)  $3(1 - 2p) \leq -4$     e)  $8(x + 4) + 2(3x - 7) < 10$     f)  $4(2x + 3) - 5(2x - 7) \geq 3(x - 7)$   
g)  $\frac{4 - 5x}{2} \geq 1$     h)  $\frac{r - 3}{5} > \frac{3r - 1}{3}$     i)  $\frac{4 - p}{3} - 9 \leq \frac{2p + 5}{5} + p$

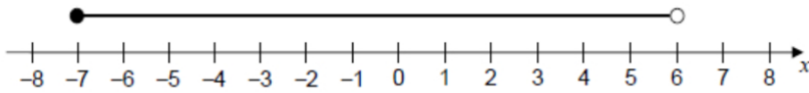
### 2. Solve these inequalities

- a)  $-2 < 3x + 1 < 13$     b)  $4 \leq 7x - 3 \leq 11$     c)  $5 < 3 - x < 8$   
d)  $2 - x \leq 5 + 2x \leq 9$     e)  $24 < 3 - 7x < 66$     f)  $3x < 5x + 1 < 2x + 9$

3. a) Write the set  $\{x \in R: -2 \leq x \leq 5\}$  using interval notation.  
b) Write the set  $\{x \in R: x > 5\}$  using interval notation.  
c) Write the set  $\{x \in R: 1 < x \leq 5\}$  using interval notation.  
d) Write the set  $\{x \in R: 6 > x \geq -4\}$  using interval notation.

e) Write the set  $\{x \in R: x \leq 1\}$  using interval notation.

4. a) Write the interval  $(0, 2]$  using set notation.  
 b) Write the interval  $[3, 5]$  using set notation.  
 c) Write the interval  $(-\infty, 2)$  using set notation.  
 d) Write the interval  $[3, \infty)$  using set notation.  
 e) Write the interval  $(-\infty, 4) \cup (6, \infty)$  using set notation.  
 f) Write the interval below using set notation.



g) Write the interval below using interval notation.



5. Given that  $3x > 25$  and  $\frac{x}{4} \leq 2\frac{3}{4}$ , list the possible integer values of  $x$ .

6. Solve the following inequalities.

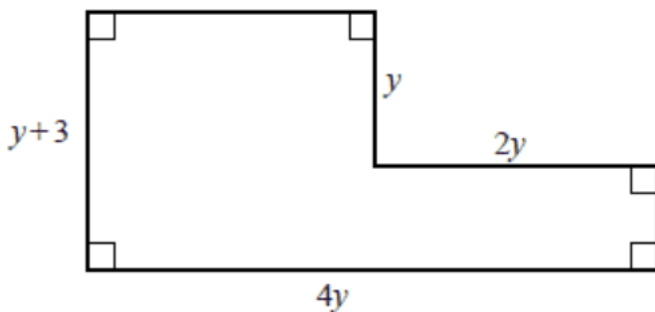
- a)  $x^2 + 4x - 12 \leq 0$       b)  $x^2 + 3x - 18 \geq 0$       c)  $2x^2 + 3x + 1 > 0$   
 d)  $3x^2 + 18 \geq 13x + 6$       e)  $x^2 < 5x + 6$       f)  $11 < x^2 + 7$   
 g)  $12 - x - x^2 > 0$       h)  $x(x - 3) \leq 4$       i)  $9 - x^2 > 0$

7. Find the range of values of  $x$  which satisfy both of the inequalities simultaneously.  
 Give your answers in interval notation.

- a)  $x^2 - 7x + 10 < 0$  and  $3x + 2 < 14$       b)  $x^2 - x - 6 > 0$  and  $7 - 3x < 14$   
 c)  $4x^2 - 3x - 1 \leq 0$  and  $4(x + 2) \leq 15 - (x + 7)$

8. a) A rectangular tile has length  $4x$  cm and width  $(x + 3)$  cm. The area of the rectangle is less than  $112$   $\text{cm}^2$ . By writing down and solving an inequality, determine the set of possible values of  $x$ .

b) A second rectangular tile of length  $4y$  cm and width  $(y + 3)$  cm has a rectangle of length  $2y$  cm and width  $y$  cm removed from one corner as shown in the diagram.



Given that the perimeter of this tile is between  $20$  cm and  $54$  cm, determine the set of possible values of  $y$ .

# SOLUTIONS TO THE EXERCISES

## Algebra

### Ex. A

- 1)  $28x + 35$     2)  $-15x + 21$     3)  $-7a + 4$     4)  $6y + 3y^2$     5)  $-4(x + 1)$     6)  $7x - 1$   
 7)  $x^2 + 5x + 6$     8)  $t^2 - 7t + 10$     9)  $6x^2 + xy - 12y^2$     10)  $4x^2 + 4x - 24$     11)  $4y^2 - 1$     12)  $12 + 17x - 5x^2$   
 13)  $x^3 + 9x^2 + 26x + 24$     14)  $x^3 - 9x^2 + 27x - 27$     15)  $6x^3 - 43x^2 - 8x - 105$     16)  $-9$

### Ex. B

- 1)  $x^2 - 2x + 1$     2)  $9x^2 + 30x + 25$     3)  $49x^2 - 28x + 4$     4)  $x^2 - 4$     5)  $9x^2 - 1$     6)  $25y^2 - 9$

### Ex. C

- 1) 7    2) 3    3)  $1\frac{1}{2}$     4) 2    5)  $-\frac{3}{5}$     6)  $-\frac{7}{3}$

### Ex. D

- 1) 2.4    2) 5    3) 1    4)  $\frac{1}{2}$

### Ex. E

- 1) 7    2) 15    3)  $\frac{24}{7}$     4)  $\frac{35}{3}$     5) 3    6) 2    7)  $\frac{9}{5}$

### Ex. F

- 1) 34, 36, 38    2) 9.875, 29.625    3) 24, 48    4)  $12x - 4$     5) a)  $30G + 180$  b) £65

### Ex G

- 1)  $x = 1, y = 3$     2)  $x = -3, y = 1$     3)  $x = 0, y = -2$     4)  $x = 3, y = 1$   
 5)  $a = 7, b = -2$     6)  $p = 11/3, q = 4/3$

### Ex H

- 1)  $x(3 + y)$     2)  $2x(2x - y)$     3)  $pq(q - p)$     4)  $3q(p - 3q)$     5)  $2x^2(x - 3)$     6)  $4a^3b^2(2a^2 - 3b^2)$   
 7)  $(y - 1)(5y + 3)$

### Ex I

- 1)  $(x - 3)(x + 2)$     2)  $(x + 8)(x - 2)$     3)  $(2x + 1)(x + 2)$     4)  $x(2x - 3)$   
 5)  $(3x - 1)(x + 2)$     6)  $(2y + 3)(y + 7)$     7)  $(7y - 3)(y - 1)$     8)  $5(2x - 3)(x + 2)$   
 9)  $(x - 3)(x - y)$     10)  $(3x + 5)(3x - 5)$     11)  $4(x - 2)(x - 1)$     12)  $(4m - 9n)(4m + 9n)$   
 13)  $y(2y - 3a)(2y + 3a)$     14)  $2(4x - 1)(x + 2)$     15)  $3(8x + 3)(x - 3)$   
 16) a)  $\frac{2(x+2)}{x-1}$     b)  $\frac{x}{x-1}$     c)  $\frac{x+2}{x}$     d)  $\frac{x}{x+5}$     e)  $\frac{x+3}{x}$   
 17) a)  $\frac{3x+4}{x+7}$     b)  $\frac{2x+3}{3x-2}$     c)  $\frac{2-5x}{2x-3}$     d)  $\frac{3x+1}{x+4}$     e)  $\frac{4(x+2)}{x-2}$

### Ex J

- 1)  $x = \frac{y+1}{7}$     2)  $x = 4y - 5$     3)  $x = 3(4y + 2)$     4)  $x = \frac{9v+20}{12}$

### Ex K

- 1)  $t = \frac{32Pr}{w}$     2)  $t = (\pm)\sqrt{\frac{32Pr}{w}}$     3)  $t = (\pm)\sqrt{\frac{3V}{\pi h}}$     4)  $t = \frac{gP^2}{2}$     5)  $t = v - \frac{Pag}{w}$     6)  $t = (\pm)\sqrt{\frac{r-a}{b}}$



**Ex L**

$$1) x = \frac{c-3}{a-b} \quad 2) x = \frac{3a+2k}{k-3} \quad 3) x = \frac{2y+3}{5y-2} \quad 4) x = \frac{ab}{b-a}$$

$$5) x = \frac{4}{3y-5} \quad 6) x = \frac{3a^2-b}{2} \quad 7) x = \frac{4}{3y^2-1} \quad 8) x = \sqrt[3]{\frac{w+2y}{y-3}}$$

**Ex M**

$$1) a) (x+2)^2 - 1 \quad b) (x-5)^2 - 28 \quad c) (x-4)^2 - 16 \quad d) (x+3)^2 - 9 \quad e) (x-1)^2 + 6$$

$$2) a) 2(x-2)^2 - 24 \quad b) 4(x-1)^2 - 20 \quad c) 3(x+2)^2 - 21$$

$$3) \quad a) 2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8} \quad b) 3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3} \quad c) 5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20} \quad d) 3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$$

$$4) (5x+3)^2 + 3 \quad 5) 9 - 2(3x+1)^2$$

**Ex. N**

$$1) a) -1, -2 \quad b) -1, 4 \quad c) -5, 3 \quad 2) a) 0, -3 \quad b) 0, 4 \quad c) 2, -2 \quad 3) a) -3/2, -2 \quad b) 1/2, 5/2 \quad c) -0.5, 4/3$$

$$4) a) 3 \pm \sqrt{13} \quad b) -4 \pm \sqrt{3} \quad c) 6 \pm \sqrt{31} \quad 5) a) -0.807, -6.19 \quad b) 6.27, -1.27 \quad c) 2.26, 0.736$$

$$6) a) x = \frac{-3 + \sqrt{89}}{10} \text{ or } x = \frac{-3 - \sqrt{89}}{10} \quad b) x = 1 + \sqrt{14} \text{ or } x = 1 - \sqrt{14}$$

$$c) x = \frac{-3 + \sqrt{23}}{2} \text{ or } x = \frac{-3 - \sqrt{23}}{2} \quad d) x = \frac{5 + \sqrt{13}}{2} \text{ or } x = \frac{5 - \sqrt{13}}{2} \quad 7) 1.23, 2.77$$

**Trigonometry****Ex. A**

$$1) a) 6.49 \text{ cm} \quad b) 6.93 \text{ cm} \quad c) 2.80 \text{ cm} \quad d) 74.3 \text{ mm} \quad e) 7.39 \text{ cm} \quad f) 6.07 \text{ cm}$$

$$2) a) 36.9^\circ \quad b) 57.1^\circ \quad c) 47.0^\circ \quad 3) 5.71 \text{ cm} \quad 4) 20.4^\circ$$

$$5) a) 45^\circ \quad b) 1 \text{ cm} \quad c) 30^\circ$$

**Ex. B**

$$1) a) 6.46 \text{ cm} \quad b) 9.26 \text{ cm} \quad c) 70.8 \text{ mm} \quad d) 9.70 \text{ cm}$$

$$2) a) 22.2^\circ \quad b) 122.9^\circ \quad 3) a) 13.7 \text{ cm} \quad b) 76.0^\circ$$

**Ex. C**

$$1) a) 4.33 \text{ cm} \quad b) 15.0 \text{ cm} \quad c) 45.2 \text{ mm}$$

$$2) a) 42.8^\circ \quad b) 52.8^\circ \quad c) 53.6^\circ$$

$$3) a) 8.13 \text{ cm} \quad b) 32.3^\circ$$

**Ex. D**

$$1) a) 18.1 \text{ cm}^2 \quad b) 18.7 \text{ cm}^2 \quad 2) 5.10 \text{ cm}$$

**Ex. E**

$$1) a) 6.29 \text{ cm} \quad b) 84.3^\circ \quad c) 5.73 \text{ cm} \quad d) 58.8^\circ \quad 2) 15.3 \text{ cm}$$

## Inequalities and Set Notation

1. a)  $x < -13$  b)  $x \leq -\frac{5}{2}$  c)  $y \geq 9$  d)  $p \geq \frac{7}{6}$  e)  $x < 2$  f)  $x \leq \frac{68}{5}$  g)  $x \leq \frac{2}{5}$   
h)  $r < -\frac{1}{3}$  i)  $p \geq -5$
2. a)  $-1 < x < 4$  b)  $1 \leq x \leq 2$  c)  $-5 < x < -2$  d)  $-1 \leq x \leq 2$  e)  $-9 < x < -3$   
f)  $-\frac{1}{2} < x < \frac{8}{3}$
3. a)  $[-2, 5]$  b)  $(5, \infty)$  c)  $(1, 5]$  d)  $[-4, 6)$  e)  $(-\infty, 1]$
4. a)  $\{x \in R : 0 < x \leq 2\}$  b)  $\{x \in R : 3 \leq x \leq 5\}$  c)  $\{x \in R : x < 2\}$   
d)  $\{x \in R : x \geq 3\}$  e)  $\{x \in R : x < 4 \text{ or } x > 6\}$  f)  $\{x \in R : -7 \leq x < 6\}$  g)  $[2, \infty)$
5. 9, 10, 11, 12, 13, 14, 15
6. a)  $-6 \leq x \leq 2$  b)  $x \leq -6 \text{ or } x \geq 3$  c)  $x > -1 \text{ or } x > -\frac{1}{2}$  d)  $x \leq \frac{4}{3} \text{ or } x \geq 3$   
e)  $-1 < x < 6$  f)  $x < -2 \text{ or } x > 2$  g)  $-4 < x < 3$  h)  $-1 \leq x \leq 4$   
i)  $-3 < x < 3$
7. a)  $(2, 4)$  b)  $(3, \infty) \cup (-7, -2)$  c)  $[-\frac{1}{4}, 0]$
8. a)  $4x(x + 3) < 112 \Rightarrow -7 < x < 4 \Rightarrow$  but lengths so  $0 < x < 4$  b)  $1.4 < y < 4.8$